Correlation between Poisson's ratio and porosity in porous materials

J. KOVÁČIK
Institute of Materials and Machine Mechanics, Slovak Academy of Sciences, Račianska 75, 831 02 Bratislava 3, Slovak Republic
E-mail: ummsjk@savba.sk

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In the case of homogeneous isotropic materials Poisson's ratio ν can be determined using modulus of elasticity E and shear modulus G as follows

$$\nu = \frac{E}{2G} - 1\cdot \tag{1}$$

In the recent works [1, 2] the percolation model was found to describe fairly well the Young's and shear modulus porosity dependence for porous materials. For that reason, Poisson's ratio porosity dependence can be simply modeled *via* Equation 1 and percolation theory, which is the main aim of this work.

The percolation equations for Young's and shear modulus porosity dependence are

$$E = E_0 \left(\frac{p_c - p}{p_c}\right)^{f_E} \quad \text{for } p \leq p_c, \tag{2}$$

$$G = G_0 \left(\frac{p_c - p}{p_c}\right)^{f_G} \quad \text{for } p \leq p_c, \tag{3}$$

where E is the effective Young's modulus and G is the effective shear modulus of porous material with porosity p, E_0 is Young's modulus and G_0 is shear modulus of solid material, p_c is a percolation threshold, i.e., the porosity at which the effective Young's and shear modulus become zero, and f_E is the characteristic exponent for the Young's modulus and f_G is the characteristic exponent for the shear modulus of porous material.

It must be pointed out that the percolation threshold for both Young's and shear modulus is considered identical on the basis of previous experimental results [1, 2]. However, the reader is requested to keep in mind that exceptions can exist. In that case you ought to consider different percolation thresholds for Young's modulus and for shear modulus.

After substituting Equations 2 and 3 in Equation 1 we obtain

$$\nu = \frac{E_0}{2G_0} \left(\frac{p_c - p}{p_c}\right)^{f_E - f_G} - 1 \quad \text{for } p \leq p_c. \tag{4}$$

Since $E_0/2G_0$ determines Poisson's ratio of solid material we can simplify the equation as follows

$$\boldsymbol{\nu} = (\boldsymbol{\nu}_0 + 1) \left(\frac{p_c - p}{p_c}\right)^{f_\nu} - 1 \quad \text{for } p \leq p_c, \tag{5}$$

where $f_\nu = f_E - f_G$ is the characteristic exponent for Poisson's ratio. $f_\nu \neq 0$, because our previous works [1, 2] showed that the characteristic exponent for Young's and shear modulus porosity dependence are usually not identical. Equation 5 represents new percolation model for the Poisson's ratio porosity dependence of isotropic and homogeneous porous solids. It implies also that the power law scaling with porosity undergoes the quantity $\nu + 1$ and not simply ν.

Usually, Poisson's ratio for porous materials is calculated according to Equation 1 from the Young's and shear modulus experimental data. Therefore, the same approach was used for the experimental data investigated in our previous works [1, 2]. Then, the Poisson's ratio data were fitted to Equation 5 with Poisson's ratio of solid material and characteristic exponent as fitting parameters. To simplify the fitting procedure the percolation threshold value was averaged from the values obtained for Young's and shear modulus data.

Again and again it is necessary to repeat the requirements that must be fulfilled prior to fitting: To model porosity dependence of Poisson's ratio, one need as wide as possible porosity range for the investigated material prepared by the same preparation method from the same type of the powder. Further, if high porosity data with no low porosity experimental data are available it is necessary to incorporate the property of the solid material into the fitting process, when known. In contrast, when no high porosity experimental data are available, it is necessary to estimate the value of percolation threshold: It seems to be preferably the apparent porosity of

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TABLE I Fitting results for Poisson's ratio porosity dependence for sintered iron [3]

Porosity range (–)	ν_0 (–)	ν_0*(–)	f_ν (–)	f_E–f_G (–)	p_c (–)	χ^2 (–)
0–0.22	0.295 ± 0.002	0.303	0.0855 ± 0.0066	0.09	≡0.41	3.3 E-4

For comparison are given also calculated values of Poisson's ratio of solid material ν_0* and characteristic exponent f_E–f_G calculated from Refs. [1, 2] (χ^2 is a minimization function).

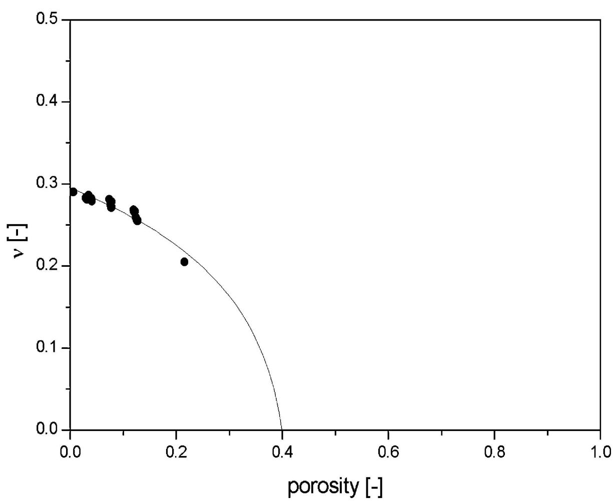

Figure 1 Poisson's ratio porosity dependence for sintered iron [3].

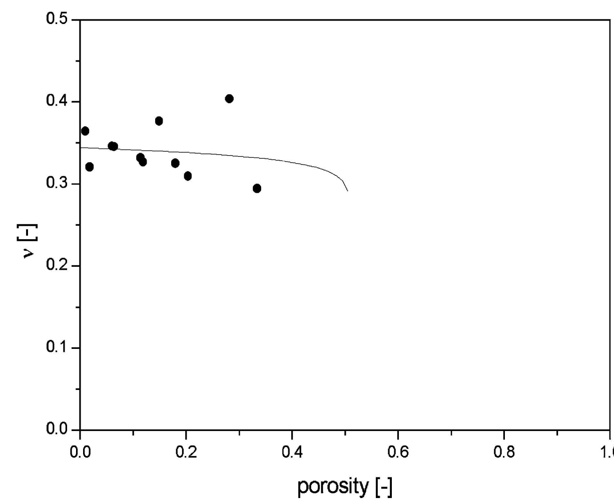

Figure 2 Poisson's ratio porosity dependence for porous ZnO [4].

the powder [2]. Only when these requirements are fulfilled one can obtain the meaningful value of the characteristic exponent and/or percolation threshold. Otherwise, the obtained values are merely the best fitting parameters valid only within the investigated porosity range.

The fitting results for sintered iron [3] (see Table I and Fig. 1) confirmed the validity of the proposed model: The porosity dependence of Poisson's ratio of sintered iron is evidently nonlinear and well described by Equation 5. There, the percolation threshold was set to 0.41 according to the results for Young's modulus in a wider porosity range [1]. Also, calculated values of Poisson's ratio of solid material ν_0* and characteristic exponent f_E–f_G from Refs. [1, 2] are in good agreement with the fitting results.

In the case of porous ZnO [4] the results indicate that the Poisson's ratio is probably independent of porosity (see Table II and Fig. 2) as points out $f_\nu \to 0$. Thus, the proposed model enables to explain why some porous materials have almost constant Poisson's ratio regardless of the porosity level. Further, it implies that in the case of identical characteristic exponents for Young's and shear modulus ($f_\nu = 0$) Poisson's ratio of porous material must be equal to the Poisson's ratio of the solid material inside the whole porosity range.

Very often the available experimental data are from low porosity range relatively far away from the percolation threshold. This problem can be overcome using the apparent porosity as the percolation threshold. When it is unknown, tap porosity can be used to investigate the effect of percolation threshold on the value of characteristic exponent for Poisson's ratio. This is the case of sintered alpha-two titanium aluminide [5] (see Fig. 3). From Table III it can be seen that using of tap porosity $p_c \equiv 0.37$ diminishes significantly the difference between fitting result and calculated value of $f_E = f_G$ from Refs. [1, 2].

It is generally accepted [1, 6, 7] that the value of percolation threshold is a function of the powder size, shape, size and shape distributions, and preparation method. Thanks to less frequently found experimental shear modulus data in the literature only powder size effect for sintered Th_2O [8] is investigated in this work: The characteristic exponent for Poisson's ratio decreases to zero with increasing powder size of the Th_2O powder (see Table IV). However, almost porosity independent Poisson's ratio for the powder size 4–44 μm was observed because the porosity range for this powder is significantly smaller than for finer ones.

Thanks to the preparation method finer powder (0–2 μm) possesses also the negative value of the Poisson's ratio (see Fig. 4). Surprisingly, this negative value is omitted

TABLE II Fitting results for Poisson's ratio porosity dependence for sintered ZnO [4]

Porosity range (–)	ν_0 (–)	ν_0* (–)	f_ν (–)	f_E–f_G (–)	p_c (–)	χ^2 (–)
0–0.33	0.344 ± 0.015	0.342	0.0087 ± 0.0239	0.01	≡0.51	1.1E-3

TABLE III Influence of fixed percolation threshold on the characteristic exponent for Poisson's ratio of sintered alpha-two titanium aluminide [5]: p_c=0.37 is tap porosity

Porosity range (–)	ν_0 (–)	ν_0^* (–)	f_ν (–)	$f_E - f_G$ (–)	p_c (–)	χ^2 (–)
0–0.30	0.343 ± 0.017	0.36	0.1044 ± 0.0220	0.27	≡0.44	6.0E–4
0–0.30	0.337 ± 0.019	0.36	0.0697 ± 0.0180	0.10	≡0.37	8.4E–4

TABLE IV Fitting results for powder size influence on the Poisson's ratio porosity dependence for porous Th_2O [8]

Th_2O powder size (μm)	Porosity range (–)	ν_0 (–)	ν_0^* (–)	f_ν (–)	$f_E - f_G$ (–)	p_c (–)	χ^2 (–)
0–2	0–0.33	0.347 ± 0.021	0.32	0.2776 ± 0.0227	0.140	≡0.375	1.3E-3
2–4	0–0.39	0.330 ± 0.013	0.32	0.1634 ± 0.0148	0.060	≡0.53	4.0E-4
4–44	0–0.27	0.314 ± 0.005	0.32	0.0854 ± 0.0056	0.130	≡0.455	3.0E-4

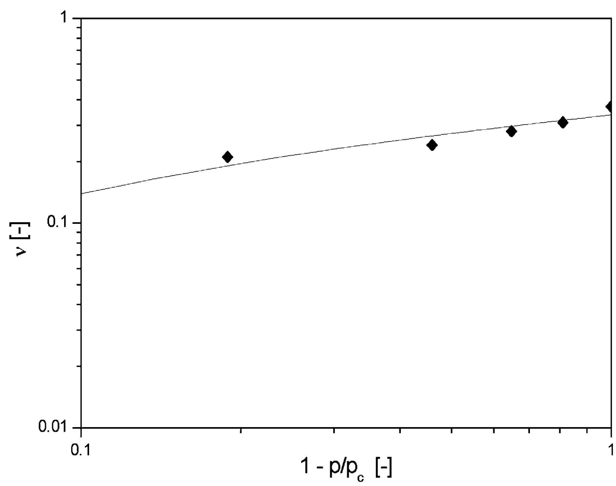

Figure 3 Log-log plot of Poisson's ratio vs. $1-p/p_c$ for sintered alpha-two titanium aluminide [5] ($p_c \equiv 0.37$ is tap porosity).

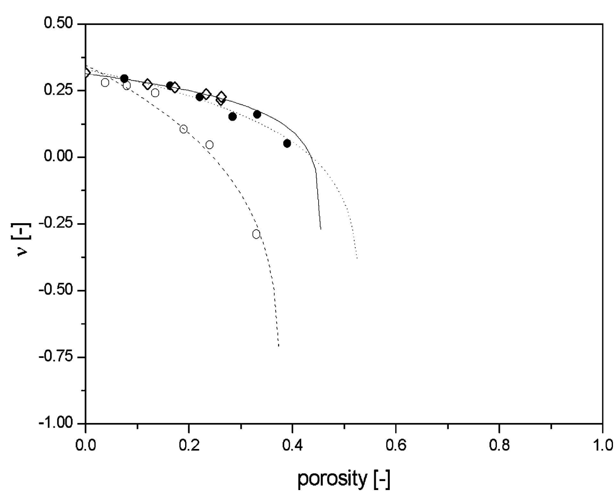

Figure 4 Influence of powder size on Poisson's ratio porosity dependence for porous Th_2O [8]: (○) 0–2 μm; (●) 2–4 μm; (◇) 4–44 μm.

in the original work of Spinner *et al*. [8] in Poisson's ratio porosity dependence plot. We were able to detect this point thanks to the corresponding points at plots of Young's and shear modulus versus porosity. How to explain this? According to the Cauchy–Hooke law for isotropic materials and as a consequence of the second law of thermodynamics the following inequality must hold for isotropic materials $-1 < \nu < 0.5$. Nevertheless, in some older literature the opinion has prevailed that the Poisson ratio should always be positive for isotropic materials (i.e., $0 < \nu < .5$).

Summarizing, the model proposed on the basis of the percolation models for Young's and shear modulus was found to describe fairly well the Poisson's ratio porosity dependence of porous materials. For the first time there is a model that is able to explain why for some porous materials porosity independent Poisson's ratio can be observed. Moreover, it is applicable also for negative values of the Poisson's ratio in the vicinity of the percolation threshold. Further, it was showed that power law scaling with porosity undergoes the quantity $\nu + 1$ and not simply Poisson's ratio.

It must be noted that all work was done under the assumption that the observed porous material is homogeneous and isotropic. When more complex stress and stiffness tensors ought to be taken into account the proposed model is either unusable or will have only limited validity.

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